The following is an appendix from:

Strindberg, S. In prep. Optimized Automated Survey Design in Wildlife Population Assessment. Unpublished PhD Thesis. University of St. Andrews.

Appendix B: Geographic Coordinates Systems & Map Projections

B.1 Approximating the Earth's Shape

To display maps, geographically referenced data located on the 3-dimensional curved surface of the earth, must be transformed onto a flat plane. This is achieved by means of a mathematical formula referred to as a projection. To simplify the mathematical calculations required for projection, the earth's highly complex surface is modelled as an ellipsoid defined in terms of its semi-major and semi-minor axes as shown in the diagram in Figure B.1. As a further simplification, sometimes a spherical representation of the earth is used instead (see Figure B.1). The difference in magnitude between the two axes of an ellipse expressed as a fraction defines the degree of ellipticity, or flattening. Values of ellipticity range between 0 and 1. The ellipticity of a sphere, whose two axes are equal, is zero. The earth is flattened at the poles and bulges out at the Equator, and has an actual ellipticity of approximately 0.003353. Thus the Earth should only be modelled as a sphere for small-scale maps, less than 1:5,000,000. At this scale the difference between a sphere and an ellipsoid cannot be detected on a map; however, to maintain accuracy for larger-scale maps (scales of 1:1,000,000 or larger), the earth should be treated as an ellipsoid (Maling, 1993).

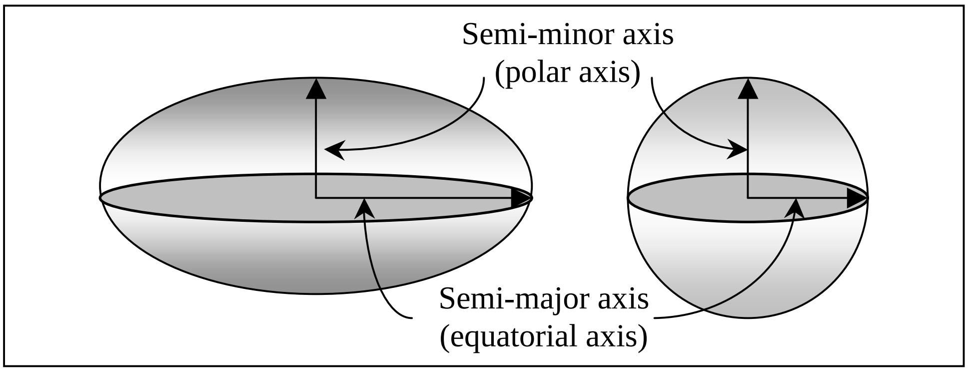

Figure B.1: Ellipsoidal (left) and spherical (right) representations of the earth.

Map projections are frequently based on ellipsoidal representations of the earth, but some projections are only supported on a sphere. It is not mathematically possible to transform data from a projection on a sphere to a projection on an ellipsoid, and simply putting the data onto an ellipsoidal representation of the earth does not increase its geodetic precision. The reverse transformation between an ellipsoid and a sphere can, however, be calculated. Using the wrong ellipsoid can result in errors in geodetic coordinates in the order of hundreds of metres. The European Petroleum Survey Group (http://www.petroconsultants.com) provides a practical guide to the various reference ellipsoids in use and the differences between them.

An ellipsoid (or sphere) together with its position relative to the centre of the earth defines a datum. A datum is a set of parameters defining a coordinate system, and a set of control points whose geometric relationships are known, either through measurement or calculation (Dewhurst, 1990). Dewhurst details how a horizontal datum provides a frame of reference for measuring locations on the surface of the earth, defining the origin and orientation of latitude and longitude lines. The coordinates of the ‘origin point’ are fixed and all other points are calculated from this control point. A local datum aligns its ellipsoid to closely fit the earth’s surface in a particular region and its ‘origin point’ is located on the surface of the earth, NAD27 and the European Datum of 1950 are examples of local datums. An earth-centred, or geocentric, datum serves as the framework for supporting locational measurement worldwide and uses the earth’s centre of mass as its origin. The most widely used datum is the World Geodetic System of 1984 (WGS-84); developed using measurements provided by satellite data to define the best earth-fitting ellipsoid. GPS measurements are based upon the WGS-84 geocentric datum.

GPS - the Global Positioning System - uses a system of 24 satellites owned by the United States Department of Defence to fix a receiver's location on the earth's surface, using the signals broadcast by the satellites. The accuracy of the fix depends on the mode of reception of the receiver. Military and other privileged users have access to the Precise Positioning System (PPS), which has an accuracy of 22 metres in the horizontal plane and 27.7 metres in the vertical plane. Civilian use is restricted to the Standard Positioning System (SPS), which has an accuracy of 100 metres horizontally and 156 metres vertically. Typically, other errors due to signal or receiver noise, such as atmospheric effects, clock errors, or reflection of signals, combine to produce an error of approximately 15 metres. Differential GPS techniques can be applied in real-time, from information broadcast over a radio link, or by post-processing GPS measurements, to obtain positional accuracy of 1-10 metres (Dana, 1999). Receivers can convert to other datums, besides the WGS-84 datum.

B.2 Coordinate Systems

Geographically referenced data is defined with respect to a coordinate system that is used to locate the data on the earth's surface. The following provides a brief description of local and global coordinate systems used not only for geo-referencing points in space, but also for navigation and geographic information systems. For a more detailed overview see Dana (1999).

The most frequently used global coordinate system is defined in terms of latitude, longitude and elevation above the reference ellipsoid. Lines of latitude are called parallels and lines of longitude are called meridians. Latitude and longitude are measured with respect to the Equator and the Greenwich Meridian, respectively. Height is measured as a perpendicular distance from the reference ellipsoid. The Earth-Centred Earth Fixed (ECEF) 3-dimensional Cartesian coordinate system has as its origin the centre of mass of the reference ellipsoid, its ***z***-axis points towards the North Pole, the ***x***-axis is defined by the intersection of the Equator and the Greenwich (Prime) Meridian, and the ***y***-axis lies 90 degrees east of the ***x***-axis, intersecting the Equator. Within the 2-dimensional Universal Transverse Mercator (UTM) coordinate system positions are expressed as the number of metres east of the central meridian plus an offset of 500 kilometres to ensure positive coordinates, and as the number of metres north of the Equator, with a 10,000 kilometres false northing for positions in the southern hemisphere. UTM zone numbers designate 6-degree longitudinal strips extending from 80 degrees south latitude to 84 degrees north latitude. UTM zone characters designate 8-degree zones extending north and south from the equator. In terms of latitudinal extent the zones are labelled C to X, with C being the southernmost zone from 80 degrees to 72 degrees south latitude. In terms of longitude, the zones are numbered 0 to 60, with zone 0 extending from 180 degrees longitude to 174 degrees west of Greenwich. The World Geographic Reference System (WGRS) is based on latitude and longitude with the globe divided into twelve bands of latitude and twenty-four zones of longitude, each 15 degrees in extent. These 15-degree areas are further divided into one-degree units identified by 15 characters. WGRS is mainly used for aircraft navigation.

Local coordinate systems are generally the national grid systems of individual countries. For example, the rectangular British National Grid, used by the Ordnance Survey of Great Britain, is based on a transverse Mercator projection with origin a point at 49 degrees north and 2 degrees west. The position of a point within the grid is given by the distance east and north of the origin, measured in metres and kilometres. Scale at the central meridian is 0.9996. In the USA, State Plane Coordinates are used. The North American Datum of 1927 (NAD27), which used the Imperial measure (feet, yards) has mainly been replaced with NAD83, based on the metre. In some countries postal codes are used as identifiers of areas. Local navigation coordinates systems, such as Loran-C, define locations by referencing measurements of electronic signals. Using time-differences, Loran-C can identify positions with an accuracy of one-quarter of a mile.

B.3 Map Projections

A map projection is used to transform 3-dimensional data on the curved surface of the earth onto a flat plane for display on a sheet of paper or a computer screen. A map projection uses mathematical formulas to relate the spherical or ellipsoidal coordinates on the globe to flat, planar coordinates. The accurate depiction of a 3-dimensional body on a 2-dimensional surface is impossible. Thus, any map projection causes distortion in the shape, area, distance, scale or direction of the data. Over a small geographic area the distortions caused by the map projection are not significant, but the selection of an appropriate map projection is crucial for larger regions. Different projections cause different types of spatial distortions. Some projections are designed to minimize the distortion of one or two of the data’s characteristics, at the expense of maximizing errors in others. Other projections attempt to only moderately distort all of these properties.

Map projections can be classified according to the property or properties that are maintained by the transformation:

<u>Conformal projections:</u> Conformal projections preserve local shape by maintaining the same scale in any direction for any map location. Scale is the relationship between distance measured on the map and the equivalent distance measured on the ground. It is not possible

to preserve the shapes of larger regions. Meridians and parallels intersect at right angles. This is accomplished by maintaining all angles. The disadvantage of this is that surface area may be greatly distorted in the process.

Equal-Area projections: Equal-Area projections preserve the area of displayed features; so all mapped areas have the same proportional relationship to the areas on the earth that they represent. This is achieved by distorting the other spatial properties of shape, angle, and scale. In equal-area projections, the meridians and parallels may not intersect at right angles. In some instances it may be difficult to distinguish an equal-area projection from a conformal projection as shapes are not obviously distorted, especially maps of smaller regions.

Equidistant projections: Equidistant maps preserve the distances between certain points. Scale is not maintained correctly by any projection throughout an entire map; however, most projections have one or more lines for which the length of the line on a map is the same length (at map scale) as the same line on the globe. Such distances are said to be true. For example, in the Sinusoidal projection, the Equator and all parallels are their true lengths. In other equidistant projections, the Equator and all meridians are true. Still others (e.g., Two-Point Equidistant) show true scale between one or two points and every other point on the map.

True-Direction projections: A map preserves direction when azimuths (bearings - angles from a point on a line to another point) are portrayed correctly in all directions. The shortest route between two points on a curved surface such as the earth is along the spherical equivalent of a straight line on a flat surface. That is the great circle on which the two points lie. True-Direction or azimuthal projections maintain some of the great-circle arcs, giving the directions or azimuths of all points on the map correctly with respect to the centre. Some true-direction projections are also conformal, or equal-area, or equidistant.

Another way of classifying map projections is to consider the earth as a transparent sphere with a light source at its centre. Consider a piece of photosensitive paper, called the projection surface, in the shape of a cylinder or cone wrapped around the earth. Let the meridians and parallels of the geographic coordinate system, as well as the outlines of the continents and oceans be projected by the light onto the paper projection surface. If the paper is then unrolled, in the first instance, the result is a map on a cylindrical projection, and in the second case on a conical projection. These two cases are shown in the diagrams in Figure B.2 below. In addition to cylindrical and conic map projections, other categories are the azimuthal and miscellaneous projections. Azimuthal projections are formed by projecting the earth's surface onto a plane, and miscellaneous projections are all those not falling into any of the other three categories.

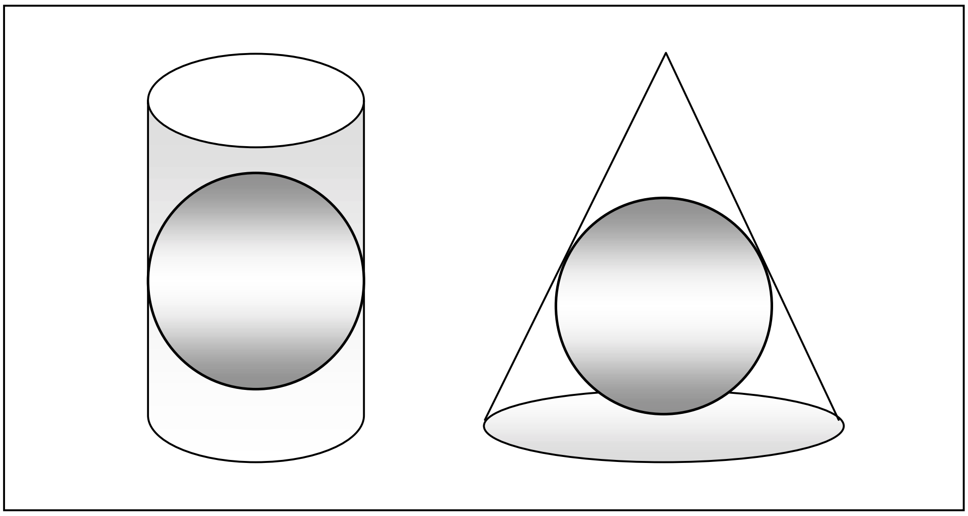

Figure B.2: Cylindrical (left) and conic (right) projections of the earth.

In addition to the distortions caused by the map projection on shape properties, errors can also result from the description of the shape of the earth, as described in section B.1. To produce accurate results for the comparison of maps at scales larger than 1:5,000,000 - even maps using the same projection - the earth's shape must be described by the same ellipsoid.

A transformation between projections is possible. This is fortunate, because a frequent problem in using Geographic Information systems (GIS) is to bring maps to a common projection. Snyder (1987) provides details of this in a comprehensive monograph. Many commercial GIS and image processing systems provide facilities for the conversion of data from one map projection to another. During such transformations care must be taken to ensure that the source and target projection use the same spherical or ellipsoidal coordinate system. Care must also be taken with the position of the standard parallels, for conic projections, and with the position of the central meridian for the cylindrical projections (Dana, 1999). Satellite images are generally not cast onto any particular projection. Additionally, they suffer from distortions caused by altitude variations, plus the effects of earth rotation during imaging. Nevertheless, a remotely sensed image can be transformed to fit a map projection, and incorporated in a GIS. This geo-referencing is most commonly achieved by defining a number of control points, which are easily and accurately located on map and image. The map and image co-ordinates of these points are used to construct two sets of polynomial equations that transform from map to image co-ordinates and vice-versa. In all instances, the choice of map projection will depend on the application.

However, there are some general guidelines in choosing a map projection. The first step in is to establish the location, size, and shape of the geographical region of interest. These three factors determine where the region to be mapped falls in relation to the distortion pattern of any projection. Maling (1993) suggests that tropical zones are best mapped using a cylindrical projection. Polar regions are best shown by an azimuthal projection, and countries or areas in the middle latitudes are best mapped using a conical projection. These global zones map into the areas in each projection where distortion is lowest. Cylindrical projections are true at the equator and distortion increases towards the poles. Conic projections are true along a chosen standard line of latitude somewhere between the pole and the equator, and distortion increases away from this standard. Azimuthal projections have a distortion pattern that increases away from the centre point, and generally distortion is worst at the edge of the map.

Unfortunately, not all regions of interest will conveniently fall into these areas of minimal distortion. Maling (1993) suggests various modifications to improve a projection's performance. Even if it is possible to minimizing distortion in general, the special properties of a projection, for a particular geographical region, still need to be considered. Each map projection has one or more standard lines, either meridians or parallels, along which there is no distortion. For a particular map-use such standard lines may be judiciously chosen, or the map may need to be conformal, equal area, or some compromise of these. In some cases, such as navigation, conformality, which maintains distance and direction for neighbouring points, is absolutely necessary. In statistical mapping, equivalence is necessary. For land cover/land use or population mapping equal-area projections are widely used. If it is necessary to consider special properties, then the final projection choice should be a function of minimized distortion and special properties.

Map projections are designed for specific purposes, and this must be kept in mind when selecting a projection. A map projection might be used for large-scale data in a limited area, while another is used for a small-scale map of the world. As well as selecting a map projection, one must consider the assumptions made about the shape of the Earth. The choice of a spherical or ellipsoidal representation of the earth and the datum used can have a significant effect on the positions of points at large scales.

B.4 Some Examples of Map Projections

This listing of some of the more frequently used map projections follows the structure and some of the descriptions given by Dana (1999).

B.4.1 Cylindrical Projections

Cylindrical Equal-Area: Cylindrical equal-area projections have straight meridians and parallels intersecting at right angles. The meridians are equally spaced, the parallels unequally spaced. There are normal, transverse, and oblique cylindrical equal-area projections. Scale is true along the central line - the equator for normal, the central meridian for transverse, and a selected line for oblique - and along two lines equidistant from the central line. Shape and scale are true along the central line, distortions increase near points 90 degrees from this line. Local angles are correct along standard parallels or standard lines, and distorted elsewhere. Recommended for narrow areas extending along the central line.

Behrmann Cylindrical Equal-Area: Behrmann's cylindrical equal-area projection uses 30 degrees north as the parallel of no distortion.

Gall's Stereographic Cylindrical: Gall's stereographic cylindrical projection results from projecting the earth's surface from the equator onto a secant cylinder (a cylinder that intersects the earth’s surface along two circles, instead of being tangentially located with respect to the globe) intersected by the globe at 45 degrees north and 45 degrees south. This projection moderately distorts distance, shape, direction, and area.

Peters: The Peters projection is a cylindrical equal-area projection that de-emphasizes area exaggerations in high latitudes by shifting the standard parallels to 45 or 47 degrees.

Mercator: The Mercator projection has straight, equally spaced meridians and parallels that intersect at right angles, just as they do on the real earth. However, this introduces distortion of area that increases towards the polar regions (the poles cannot be shown). Scale is true at the equator or at two standard parallels equidistant from the equator. The projection is often used for marine navigation because all straight lines on the map are lines of actual compass bearing. Other directional uses for this projection include air travel, wind direction, ocean currents, and conformal world maps. The best use of this projection's conformal properties applies to regions near the Equator, such as Indonesia and parts of the Pacific Ocean.

Miller Cylindrical: The Miller projection has straight meridians and parallels that meet at right angles, but straight lines are not of constant azimuth. Shape and area distortions increases from the Equator toward the poles, with minimally distortions between the 45th parallels. Directions and local angles are true only along the Equator. The projection avoids the scale exaggerations of the Mercator map at the polar regions. This is accomplished by reducing the distance between parallels as they approach the poles, but introduces distortion in local shape and direction. The Miller projection is only useful as a general-purpose world map.

Oblique Mercator: Oblique Mercator projections are used to portray regions along great circles. Distances are true along a great circle defined by the tangent line formed by the sphere and the oblique cylinder; elsewhere distance, shape, and areas are distorted. Once used to map Landsat images (now replaced by the Space Oblique Mercator), this projection is used for areas that are long, thin zones at a diagonal with respect to north (Large-scale mapping of Switzerland, Borneo, Madagascar, or the Alaskan panhandle, for example). This projection is sometimes referred to as the Hotine Oblique Mercator projection.

Transverse Mercator: Transverse Mercator projections result from projecting the sphere onto a tilted cylinder tangent to a central meridian. Transverse Mercator maps are often used to portray areas with larger north-south than east-west extent. Although, local scale is maintained, distortion of scale, distance, direction and area increase away from the central meridian. Global projection becomes infinite 90 degrees from the central meridian. Use should be limited to 15 to 20 degrees on both sides of the central meridian. Many national grid systems, such as the British National Grid (BNG), are based on the Transverse Mercator projection. The projection is also known as the Gauss-Kruger projection or the Universal Transverse Mercator Projection (UTM).

Plate Carree or Equidistant Cylindrical: The Plate Carree projection forms a grid of rectangles of equal size, shape, and area. The polar regions are less distorted in scale and area than they are with the Mercator projection. The meridians and parallels intersect at right angles. The traditional Plate Carree projection uses the equator as its central parallel, but any line may be used. In the first instance the grid cells are perfect squares otherwise they become rectangular. Distortions in shape and area increase with distance from the standard parallels. General directions are distorted, except locally along the standard parallels. Scale is correct along all the meridians and along the standard parallels. Best used for maps of small areas or for simple portrayals of the world or regions with minimal geographic data, such as index maps.

Cassini: This transverse cylindrical projection is analogous to the Plate Carree projection in the same way that the Transverse Mercator is to the Mercator projection. It maintains scale along the central meridian and all lines parallel to it, and is neither equal-area nor conformal, but a compromise of both features. It is most suited for large scale mapping of areas near the central meridian with predominantly north-south in extent. Transverse Mercator is often preferred because of difficulty in measuring scale and direction on Cassini.

B.4.2 Pseudocylindrical Projections

Pseudocylindrical projections resemble cylindrical projections, with straight and parallel latitude lines and equally spaced meridians, but the meridians are curves.

Mollweide: The Mollweide projection, used for world maps, is pseudocylindrical and equal-area. The central meridian is straight. The 90th meridians are circular arcs. Parallels are straight, but unequally spaced. Scale is true only along the standard parallels of 40° 44' north and 40° 44' south, and distortion increases with distance from these lines, becoming severe at the edges of the projection. Local angles and shapes are correct only at the intersection of the central meridian and these standard parallels. The Mollweide projection is also known as the Babinet, Elliptical, Homolographic, or Homalographic projection.

Eckert Projections: The Eckert IV projection, used for world maps, is a pseudocylindrical and equal-area. The central meridian is straight, the 180th meridians are semi-circles, and other meridians are elliptical. Scale is true along the parallel at 40:30 degrees north and south. The Eckert VI Equal-Area, used for maps of the world, is pseudocylindrical and equal area. The central meridian and all parallels are at right angles; all other meridians are sinusoidal curves. Shape distortion increases at the poles. Scale is correct at standard parallels of 49:16 degrees north and south.

Robinson: The Robinson projection is based on tabular coordinates, rather than mathematical formulae. The meridians are equally spaced, resembling elliptical arcs, concave toward the central meridian. The projection distorts shape, area, scale, and distance in an attempt to balance the errors of projection properties. Distortions of shape and area are minimized within 45 degrees of the origin and along the Equator. Directions are generally distorted and scale is usually made true along latitudes 38 degrees north and south. It was developed for use in general and thematic world maps.

Sinusoidal Equal-Area: Sinusoidal equal-area maps have straight parallels at right angles to a central meridian. Other meridians are sinusoidal curves. Scale is true only on the central meridian and the parallels. Local angles are shapes are correct along the central meridian and the Equator. Often used in countries with a larger north-south than east-west extent, especially for regions near the Equator. Distortion along outer meridians can be reduced by interrupting the continuity of the projection over the oceans and by recentering the continents around their own central meridians.

B.4.3 Conic Projections

Equidistant Conic: This is the simplest conic projection. The space between each meridian is equal, as is the space between each of the concentric arcs that represent the parallels. The poles are represented as arcs rather than points. Local shapes and local directions are true along the standard parallels. Shape and area distortions are constant along any given parallel, and the distortion increases with distance from the standard parallels. Distance is true along the meridians and the standard parallels. Scale is constant along any given parallel, but changes from parallel to parallel. The Equidistant Conic projection is used for regional mappings at the mid-latitude for regions with a predominantly east-west extent. The range in latitude should not exceed 30 degrees.

Albers Equal-Area Conic: This conic projection uses two standard parallels to reduce some of the distortion produced when only one standard parallel is used. However, distortions of scale and distance still occur, except along the standard parallels. Areas are proportional and direction, area, and shape are distorted away from standard parallels. These distortions are minimized in the region between the standard parallels. The 90-degree angles between meridians and parallels are preserved. Used for portrayals of areas near to, but on one side of, the equator that extend more in the east-west orientation than those lying north-south, such as the United States, for example. Total range in latitude from north to south should not exceed 30 - 35 degrees. No Limitations on east to west range.

Lambert Conformal Conic: This conic projection is similar to the Albers Equal-Area Conic projection, except that it portrays shape more accurately than area. It is normally based on two standard parallels. Meridians and parallels intersect at right angles, but the spacing between lines of latitude increases beyond the standard parallels. The poles are represented as a single point. Local angles and shapes are maintained. Area and shape are distorted away from standard parallels. Scale is correct along the standard parallels. Area and distance scales are reduced between standard parallels and increased beyond them. This projection is one of the best for middle latitudes for regions with an east-west in extent, and is frequently used for maps of North America. Total range in latitude should not exceed 35 degrees.

Polyconic: The Polyconic projection was used for most of the earlier United States Geological Society topographic quadrangles. The projection is based on an infinite number of cones tangent to an infinite number of parallels. The central meridian is straight. Other meridians are complex curves. The parallels are non-concentric circles. Scale is true along each parallel and along the central meridian.

B.4.4 Azimuthal Projections

Azimuthal Equidistant: With the Azimuthal equidistant projection, the world is projected onto a flat surface from any centre point on the globes. Distances and directions measured from the centre are true. Distortion of other properties increases away from the centre point. This projection can accommodate all aspects, equatorial, polar and oblique. This projection is sometimes used to show sea- and air-route distances. Although the entire globe can be projected, its use is generally limited to 90 degrees from the centre. Polar-aspect projections are best for regions within a 30-degree radius because in this case, there is only minimal distortion (Roblin, 1969).

Lambert Azimuthal Equal-Area: The Lambert azimuthal equal-area projection is sometimes used to map large ocean areas. The central meridian is a straight line, while other meridians are curved. A straight line drawn through the centre point is on a great circle.

Orthographic: Orthographic projections are used for perspective views of hemispheres. Area and shape are distorted. Distances are true along the equator and other parallels.

Stereographic: Stereographic projections are used for navigation in polar regions. Directions are true from the centre point and scale increases away from this point, as does distortion in area and shape. Local angles are accurate everywhere. All meridians and parallels are shown as circular arcs or straight lines, and they intersections are 90 degrees. In the equatorial aspect, the parallels curve in opposite directions on either side of the Equator. The parallel opposite in sign to the central latitude is a straight line; other parallels are concave toward the pole on the same side of the straight parallel. The Stereographic projection is normally limited to one hemisphere, a radius of 90 degrees from the centre point.

B.4.5 Miscellaneous Projections

Unprojected Maps: Unprojected maps include those that are formed by considering longitude and latitude as a simple rectangular coordinate system. Scale, distance, area, and shape are all distorted with the distortion increasing toward the poles.

Space Oblique Mercator: The Space Oblique Mercator is a projection designed to show the curved ground-track of Landsat images. There is little distortion along the ground-track but only within the narrow band (about 15 degrees) of the Landsat image.

References

Dana, P. H. 1999. *The Geographer's Craft*. Department of Geography, University of Colorado at Boulder (http://www.colorado.edu/geography/gcraft/contents.html).

Dewhurst, W. T. 1990. *The Application of Minimum-curvature-derived Surfaces in the Transformation of Positional Data from the North American Datum of 1927 to the North American Datum of 1983*. NOAA Technical Memorandum NOS NGS-50. Rockville, USA.

Maling, D.H. 1993. *Coordinate Systems and Map Projections* (2nd ed.). Pergamon Press, Oxford, UK.

Roblin, H. S. 1969. *Map Projections*. Fletcher & Son, Norwich, UK.

Snyder, J. P. 1987. *Map Projections - A Working Manual*. USGS Professional Paper 1395. US Government Printing Office, Washington DC.